Slepian-Based Two-Dimensional Estimation of Time-Frequency Variant MIMO-OFDM Channels

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***Abstract*—A linear channel estimator for multiple-input multiple-output orthogonal frequency-division multiplexing (MIMO-OFDM) systems, based on a two-dimensional Slepian expansion, is presented. The estimator is meant to be part of an iterative receiver. We consider both estimation based on pilots only and on pilots and data, the latter considered as a reference for the case when feedback from decoders is exploited. Performances are analyzed via computer simulations comparing the relative minimum square error (RMMSE) of an analogous one-dimensional estimator and the proposed extension.**

***Index Terms*—Channel estimation, discrete prolate spheroidal sequences, iterative receivers, multiple-input multiple-output (MIMO) channels, orthogonal frequency-division multiplexing (OFDM) systems.**

I. Introduction

WIRELESS high-data-rate system design is a hot topic in current research and orthogonal frequency-division multiplexing (OFDM) [1] coupled with multiple-input multiple-output (MIMO) channels [1] and iterative receivers [2] represents a very attractive choice. In such systems, channel state information at the receiver is fundamental to allow efficient use of coherent modulation. Channel estimation for OFDM systems has been proposed via singular value decomposition [3] exploiting frequency correlation, and via two-dimensional Wiener filtering [4] exploiting time and frequency correlations. Robustness to channel-statistics mismatch is analyzed in [5], while complexity issues has been taken into account also via parametric channel modeling [6].

Recently, exploiting the work of Slepian [7] on discrete prolate spheroidal (DPS) sequences, a robust low-complexity channel estimator has been proposed [8] and applied in the iterative receiver for multi-carrier code-division multiple access systems [2] and for MIMO-OFDM systems [9]. Work in [8] has been extended in [10] and [11] to account for both time and frequency variations via use of multidimensional DPS sequences [12], [13]. In this letter, we design a Slepian-based estimator intended for MIMO-OFDM systems with iterative receivers, proposing the extension of [9] to the two-dimensional (time and frequency) case. The benefits of such an approach are: 1) robustness—no assumption on channel statistics is needed but knowledge of the maximum delay spread and maximum Doppler spread; 2) low complexity—fewer coefficients to be estimated due to the concentration of the space; and 3) accuracy—joint two-dimensional processing exploits both time and frequency correlations.

As we focus on channel estimation, all the transmitted symbols are assumed to be known at the receiver. Two cases are considered, and the receiver uses: 1) only pilots, 2) both pilots and data. In a real iterative receiver, only pilot symbols are available at the first iteration, while soft estimates from the decoder are available at successive iterations to replace data symbols. Soft estimates will converge, in a well-designed receiver, to the correct values of data symbols; thus, the performance of the channel estimator will stay between performance of the two considered cases, starting from the former and approaching the latter iteratively.

As for pilot placement, the proposed estimator allows, due to the two-dimensional processing, pilot patterns different from block-type (for some given time slots, all subcarriers contain pilots) and comb-type (for some given subcarriers, all time slots contain pilots). Pilots may sparsely sample the time-frequency domain as long as they obey the limits of the samples theorem set by delay spread and Doppler spread.

The rest of this letter is organized as follows: Section II introduces the system model; the channel estimator is presented in Section III; Section IV shows the performance obtained via computer simulation in terms of relative minimum mean square error (RMMSE) versus signal-to-noise ratio (SNR); and some concluding remarks are given in Section V.

Notation—Column vectors (resp. matrices) are denoted with lowercase (resp. uppercase) bold letters; a_n (resp. $A_{n,m}$) denotes the nth (resp. (n,m)th) element of vector $\boldsymbol{a}$ (resp. matrix $\boldsymbol{A}$); $\mathrm{diag}(\boldsymbol{a})$ denotes a diagonal matrix whose main diagonal is $\boldsymbol{a}$; $\boldsymbol{I}_N$ denotes the $N \times N$ identity matrix; $\boldsymbol{e}_N$ denotes a vector of length N whose elements are 1; $\mathbb{E}\{.\}$, $(.)^*$, $(.)^{\mathrm{T}}$, and $(.)^{\mathrm{H}}$ denote expectation, conjugate, transpose, and conjugate transpose operators; $\delta_{n,m}$ is the Kronecker delta; $\otimes$ denotes the Kronecker product; $\lceil a \rceil$ denotes the smallest integer value greater than or equal to a; j denotes the imaginary unit; $\mathcal{N}_{\mathbb{C}}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ denotes a circular symmetric complex normal distribution with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$; and the symbol $\sim$ means "distributed as."

II. Analytical Model

We consider a MIMO-OFDM system with K transmit antennas and N receive antennas. For data transmission, each transmit antenna adopts OFDM with M subcarriers. Data are assumed to be encoded within a frame composed of S OFDM blocks, with each OFDM block composed of M symbols.

In the following, referring to the mth subcarrier during transmission of the sth OFDM block for the generic frame, $x_k[m,s]$

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denotes the (*Frequency Domain*) symbol[1] transmitted by the kth transmit antenna; $H_{n,k}[m,s]$ denotes the (*Frequency Domain*) channel coefficient between the kth transmit antenna and the nth receive antenna; $r_n[m,s]$ denotes the (*Frequency Domain*) received signal at the nth receive antenna; and $w_n[m,s]$ the corresponding additive noise contribution. We denote the transmitted vector, the channel matrix, the noise vector, and the received vector as

$$\begin{aligned}\boldsymbol{x}[m,s] &= (x_1[m,s],\dots,x_K[m,s])^{\mathrm{T}}\\ \boldsymbol{H}[m,s] &= \begin{pmatrix} H_{1,1}[m,s] & \dots & H_{1,K}[m,s]\\ \vdots & \ddots & \vdots\\ H_{N,1}[m,s] & \dots & H_{N,K}[m,s]\end{pmatrix}\\ \boldsymbol{w}[m,s] &= (w_1[m,s],\dots,w_N[m,s])^{\mathrm{T}} \sim \mathcal{N}_{\mathbb{C}}\left(\boldsymbol{0},\sigma_w^2\boldsymbol{I}_N\right)\\ \boldsymbol{r}[m,s] &= (r_1[m,s],\dots,r_N[m,s])^{\mathrm{T}}.\end{aligned}$$

Assuming the cyclic-prefix length exceeds the channel delay spread, the discrete-time model for the received signal is

$$\boldsymbol{r}[m,s] = \boldsymbol{H}[m,s]\boldsymbol{x}[m,s] + \boldsymbol{w}[m,s]. \tag{1}$$

It is worth noticing that m and s represent frequency-variation and time-variation, respectively. The channel is considered time-variant meaning that it does not remain constant within the frame: different OFDM blocks experience different attenuations, and different subcarriers within the same OFDM block experience different correlated attenuations.

A. Slepian Expansion

We consider a wireless channel with maximum normalized delay spread $\eta_{\max}^{(\mathrm{d})}$ and maximum normalized Doppler spread $\nu_{\max}^{(\mathrm{D})}$, i.e., for each transmit/receive antennas pair, $[-\eta_{\max}^{(\mathrm{d})}, +\eta_{\max}^{(\mathrm{d})}] \times [-\nu_{\max}^{(\mathrm{D})}, +\nu_{\max}^{(\mathrm{D})}]$ is the rectangular support of the scattering function

$$\mathcal{H}_{n,k}(\eta,\nu) = \sum_{m=1}^{M}\sum_{s=1}^{S} H_{n,k}(m,s)\exp\left(-j2\pi(\eta m + \nu s)\right).$$

Variables η and ν represent time and frequency as they correspond via a Fourier transformation to frequency index m and time index s, accounting for delay and Doppler, respectively.

Let $v_\ell[m]$ and $\lambda_\ell^{(\mathrm{d})}$ denote the mth sample of the ℓth DPS sequence and the corresponding eigenvalue, for the interval $m = 1,\dots,M$ and bandwidth extension $\eta_{\max}^{(\mathrm{d})}$; and $u_i[s]$ and $\lambda_i^{(\mathrm{D})}$ the sth sample of the ith DPS sequence and the corresponding eigenvalue, for the interval $s = 1,\dots,S$ and bandwidth extension $\nu_{\max}^{(\mathrm{D})}$; defined as the solutions to

$$\sum_{m'=1}^{M} 2\eta_{\max}^{(\mathrm{d})}\mathrm{sinc}\left(2\eta_{\max}^{(\mathrm{d})}(m'-m)\right)v_\ell[m'] = \lambda_\ell^{(\mathrm{d})}v_\ell[m]$$

$$\sum_{s'=1}^{S} 2\nu_{\max}^{(\mathrm{D})}\mathrm{sinc}\left(2\nu_{\max}^{(\mathrm{D})}(s'-s)\right)u_i[s'] = \lambda_i^{(\mathrm{D})}u_i[s].$$

The DPS sequences have resulted the bandlimited sequences simultaneously most concentrated in a finite time interval [7].

As they distinguish the two dimensions of the wireless channel,[2] $v_\ell[m]$ and $u_i[s]$ will be denoted frequency-DPS (f-DPS) and time-DPS (t-DPS) sequences, respectively. We consider the following two-dimensional Slepian expansion (making use of an orthogonal basis based on DPS sequences):

$$H_{n,k}(m,s) \approx \sum_{\ell=1}^{L}\sum_{i=1}^{I}\psi_{n,k}[\ell,i]u_i[s]v_\ell[m] \tag{2}$$

where $\psi_{n,k}[\ell,i]$ is the (ℓ,i)th Slepian coefficient for the link between the kth transmit antenna and the nth receive antenna, $M^{(\mathrm{d})} \le L \le M$ and $S^{(\mathrm{D})} \le I \le S$, being $M^{(\mathrm{d})} = \lceil 2\eta_{\max}^{(\mathrm{d})}M\rceil + 1$ and $S^{(\mathrm{D})} = \lceil 2\nu_{\max}^{(\mathrm{D})}S\rceil + 1$ the approximate signal space extensions. The reason behind the concentration of the space, along both delay and Doppler dimensions, is that the eigenvalues $\lambda_\ell^{(\mathrm{d})}$ (resp. $\lambda_i^{(\mathrm{D})}$) rapidly become negligible for $\ell > 2\eta_{\max}^{(\mathrm{d})}S$ (resp. $i > 2\nu_{\max}^{(\mathrm{D})}S$) [7].

In the following, $\boldsymbol{v}[m] = (v_1[m],\dots,v_L[m])^{\mathrm{T}}$ is the vector collecting the values of the f-DPS sequences for a given frequency, and $\boldsymbol{\lambda}^{(\mathrm{d})} = (\lambda_1^{(\mathrm{d})},\dots,\lambda_L^{(\mathrm{d})})^{\mathrm{T}}$ is the vector collecting the corresponding eigenvalues; $\boldsymbol{u}[s] = (u_1[s],\dots,u_I[s])^{\mathrm{T}}$ is the vector collecting the values of the t-DPS sequences for a given time, and $\boldsymbol{\lambda}^{(\mathrm{D})} = (\lambda_1^{(\mathrm{D})},\dots,\lambda_I^{(\mathrm{D})})^{\mathrm{T}}$ is the vector collecting the corresponding eigenvalues. Also we denote

$$\begin{aligned}\boldsymbol{\varphi}[m,s] &= \boldsymbol{v}[m]\otimes\boldsymbol{u}[s]\\ \boldsymbol{\Lambda} &= \frac{1}{2\eta_{\max}^{(\mathrm{d})}}\frac{1}{2\nu_{\max}^{(\mathrm{D})}}\mathrm{diag}\left(\boldsymbol{\lambda}^{(\mathrm{d})}\otimes\boldsymbol{\lambda}^{(\mathrm{D})}\right).\end{aligned}$$

B. Signal Model

The signal model for channel estimation is

$$\boldsymbol{r} = \Xi\boldsymbol{\psi} + \boldsymbol{w} \tag{3}$$

where $\boldsymbol{r}$, Ξ, and $\boldsymbol{w}$ collect the received signals, the transmitted signals, and the noise contribution, respectively, for all transmit antennas, receive antennas, subcarriers, and OFDM blocks, and $\boldsymbol{\psi}$ collects the appropriate Slepian coefficients. More specifically, for the received signals and the noise

$$\begin{aligned}\boldsymbol{r} &= \left(\boldsymbol{r}^{\mathrm{T}}[1],\dots,\boldsymbol{r}^{\mathrm{T}}[S]\right)^{\mathrm{T}}\\ \boldsymbol{r}[s] &= \left(\boldsymbol{r}^{\mathrm{T}}[1,s],\dots,\boldsymbol{r}^{\mathrm{T}}[M,s]\right)^{\mathrm{T}}\\ \boldsymbol{w} &= \left(\boldsymbol{w}^{\mathrm{T}}[1],\dots,\boldsymbol{w}^{\mathrm{T}}[S]\right)^{\mathrm{T}}\\ \boldsymbol{w}[s] &= \left(\boldsymbol{w}^{\mathrm{T}}[1,s],\dots,\boldsymbol{w}^{\mathrm{T}}[M,s]\right)^{\mathrm{T}}.\end{aligned}$$

Also, for the transmitted signals and the Slepian coefficients

$$\begin{aligned}\Xi &= \left(\Xi^{\mathrm{T}}[1],\dots,\Xi^{\mathrm{T}}[S]\right)^{\mathrm{T}}\\ \Xi[s] &= \left(\Xi^{\mathrm{T}}[1,s],\dots,\Xi^{\mathrm{T}}[M,s]\right)^{\mathrm{T}}\\ \Xi[m,s] &= \boldsymbol{I}_N\otimes(\boldsymbol{x}[m,s]\otimes\boldsymbol{\varphi}[m,s])^{\mathrm{T}}\\ \boldsymbol{\psi} &= \left(\boldsymbol{\psi}_1^{\mathrm{T}},\dots,\boldsymbol{\psi}_N^{\mathrm{T}}\right)^{\mathrm{T}}\\ \boldsymbol{\psi}_n &= \left(\boldsymbol{\psi}_{n,1}^{\mathrm{T}},\dots,\boldsymbol{\psi}_{n,K}^{\mathrm{T}}\right)^{\mathrm{T}}\\ \boldsymbol{\psi}_{n,k} &= \left(\boldsymbol{\psi}_{n,k}^{\mathrm{T}}[1],\dots,\boldsymbol{\psi}_{n,k}^{\mathrm{T}}[L]\right)^{\mathrm{T}}\\ \boldsymbol{\psi}_{n,k}[\ell] &= (\psi_{n,k}[\ell,1],\dots,\psi_{n,k}[\ell,I])^{\mathrm{T}}.\end{aligned}$$

Comparison shows the equivalence between (1) and (3).

[1]Both for pilot and data symbols.

[2]Frequency and time, or, equivalently, delay and Doppler.

III. Channel Estimator

Restricting our attention to linear channel estimators, we have the following estimator (see the Appendix for details):

$$\begin{aligned}\boldsymbol{\psi} &= \boldsymbol{C}_\psi \hat{\boldsymbol{\Xi}}^{\mathrm{H}} (\hat{\boldsymbol{\Xi}} \boldsymbol{C}_\psi \hat{\boldsymbol{\Xi}}^{\mathrm{H}} + \boldsymbol{\Delta})^{-1} \boldsymbol{r} \\ &= \left(\hat{\boldsymbol{\Xi}}^{\mathrm{H}} \boldsymbol{\Delta}^{-1} \hat{\boldsymbol{\Xi}} + \boldsymbol{C}_\psi^{-1}\right)^{-1} \hat{\boldsymbol{\Xi}}^{\mathrm{H}} \boldsymbol{\Delta}^{-1} \boldsymbol{r} \end{aligned} \quad (4)$$

to be used in (2), where $\boldsymbol{C}_\psi = \mathbb{E}\{\boldsymbol{\psi}\boldsymbol{\psi}^{\mathrm{H}}\} = \boldsymbol{I}_{(NK)} \otimes \boldsymbol{\Lambda}$; it denotes the covariance matrix of the Slepian coefficients; $\hat{\boldsymbol{\Xi}} = \mathbb{E}\{\boldsymbol{\Xi}\}$, it contains the expected transmitted symbols;[3] and $\boldsymbol{\Delta} = \boldsymbol{\Theta} + \sigma_w^2 \boldsymbol{I}_{(NMS)}$, where

$$\begin{aligned}\boldsymbol{\Theta} &= \mathrm{diag}(\boldsymbol{\vartheta}) \\ \boldsymbol{\vartheta} &= \left(\boldsymbol{\vartheta}^{\mathrm{T}}[1], \ldots, \boldsymbol{\vartheta}^{\mathrm{T}}[S]\right)^{\mathrm{T}} \\ \boldsymbol{\vartheta}[s] &= \left(\boldsymbol{\vartheta}^{\mathrm{T}}[1,s], \ldots, \boldsymbol{\vartheta}^{\mathrm{T}}[M,s]\right)^{\mathrm{T}} \\ \boldsymbol{\vartheta}[m,s] &= \left(\sum_{k=1}^{K} \left(1 - |\hat{x}_k[m,s]|^2\right)\right) \boldsymbol{e}_N. \end{aligned}$$

It is worth noticing that the matrix inversion lemma in (4) replaces the inversion of an $NMS \times NMS$ matrix with the inversion of an $NKLI \times NKLI$ matrix, saving computations when $K < 1/(2\eta_{\max}^{(\mathrm{d})} 2\nu_{\max}^{(\mathrm{D})})$. Also it is worth noticing that both $\boldsymbol{C}_\psi$ and $\boldsymbol{\Delta}$ are diagonal; thus, their inversion is not computationally prohibitive. More specifically, when both pilots and data symbols are known, $\boldsymbol{\Theta}$ is the null matrix.

Channel estimation is evaluated via RMMSE

$$\delta_H = \frac{\mathbb{E}\left\{\left|H_{n,k}[m,s] - \hat{H}_{n,k}[m,s]\right|^2\right\}}{\mathbb{E}\left\{|H_{n,k}[m,s]|^2\right\}}.$$

For pilot placement, we simply consider pilots equivalent to samples for the sampling problem. Optimal pilot placement [14] falls beyond the scope of this letter. Let $\Omega^{(\mathrm{d})}$ (resp. $\Omega^{(\mathrm{D})}$) denote the spacing between two consecutive pilots along the frequency (resp. time) dimension, and then according to the sampling theorem, the following conditions must be fulfilled: $\Omega^{(\mathrm{d})} < 1/2\eta_{\max}^{(\mathrm{d})}$, $\Omega^{(\mathrm{D})} < 1/2\nu_{\max}^{(\mathrm{D})}$. Typically, $\Omega^{(\mathrm{d})} = 1/4\eta_{\max}^{(\mathrm{d})}$ and $\Omega^{(\mathrm{D})} = 1/4\nu_{\max}^{(\mathrm{D})}$ represents a good compromise between complexity and accuracy; thus, the number of pilots along the frequency dimension (M_p) and the number of pilots along the time dimension (S_p) are

$$M_p \approx 4\eta_{\max}^{(\mathrm{d})} M, \quad S_p \approx 4\nu_{\max}^{(\mathrm{D})} S. \quad (5)$$

Also, it is worth noticing that in a two-dimensional space, there is more freedom for the choice of the sampling grid than in a one-dimensional space in which we only have the rectangular grid. Samples along the horizontal dimension may be shifted when referring to different vertical sample points, and vice versa. Denoting R as the shift that pilot indexes in time at a given subcarrier have with respect to the pilot indexes in time at the previous subcarrier, then the grid $\mathcal{G}_{M,S}(M_p, S_p, R)$ represents the following set of indexes (for pilot placement):

$$\mathcal{G}_{M,S}(M_p,S_p,R) = \Big\{(m,s) \in \mathbb{N}_M \times \mathbb{N}_S : m \in \mathcal{G}_M^{(\mathrm{d})}(M_p) \\ s \in \mathcal{G}_S^{(\mathrm{D})}(S_p,R,m)\Big\}$$

[3]A turbo receiver computes expectation by soft estimates from the decoder.

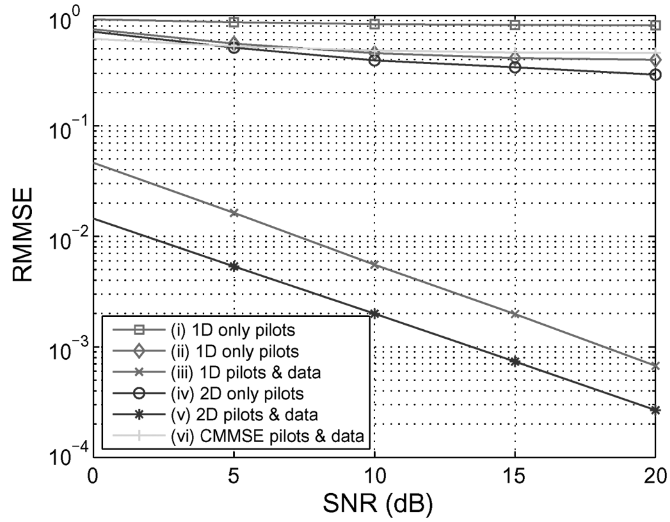

Fig. 1. Performance of a 2×2 system with $M = 32$, $S = 128$, and $\eta_{\max}^{(\mathrm{D})} = 0.08$, $\nu_{\max}^{(\mathrm{D})} = 0.005$.

with $\mathcal{G}_M^{(\mathrm{d})}(M_p) = \{\lceil (2m-1)M/2M_p \rceil\}_{m=1}^{M_p}$, and with $\mathcal{G}_S^{(\mathrm{D})}(S_p,R,m) = \{\mathrm{mod}(\lceil (2s-1)S/2S_p \rceil + (m-1)R, S)\}_{s=1}^{S_p}$. Better performance have been achieved with $R \approx \Omega^{(\mathrm{D})}/2$, coherently with the fact that hexagonal sampling outperforms rectangular sampling in two-dimensional domains [15].

IV. Simulation Results

Computer simulations for MIMO-OFDM systems with coherent binary phase shift keying (BPSK) modulation have been performed to obtain RMMSE-versus-SNR performance when the receiver makes use of: 1) only pilot symbols, 2) both pilot and data symbols. In the first case, transmitted symbols corresponding to data symbols are replaced with 0s and transmitted symbols corresponding to pilot symbols are kept with their correct values. The number of pilots is defined according to (5) and hexagonal grid is assumed. In the second case, transmitted symbols corresponding to both data and pilot symbols are kept with their correct values.

OFDM with $M = 32$ subcarriers (thus, 32 BPSK symbols per OFDM block) and $S = 128$ OFDM blocks per frame, and MIMO channels with $K = 2$ transmit antennas and $N = 2$ receive antennas have been considered. Results refer to synthetic channels that simulate Rayleigh fading according to Jakes' model [16]. Channel coefficients for each transmit-receive antenna pair have been generated according to a model with maximum normalized delay spread $\eta_{\max}^{(\mathrm{d})} = 0.08$ and maximum normalized Doppler frequency $\nu_{\max}^{(\mathrm{D})} = 0.005$. The signal space concentration is a reduction from $(M,S) = (32,128)$ to $(M^{(\mathrm{d})}, S^{(\mathrm{D})}) = (7,3)$, and $L \times I = 10 \times 6 = 60$ coefficients have been used for the Slepian expansion.[4]

Fig. 1 compares the performance of: 1) one-dimensional estimator [9] using only pilots according to $M_p = 10$, $S_p = 3$,

[4]This scenario represents, for a system operating at 910 MHz, an environment whether with maximum delay of 3 μs and maximum velocity of 5 km/h, or with maximum velocity of 80 km/h and maximum delay of 0.2 μs, thus corresponding to pedestrians in an urban area or vehicles in a rural area [17].

$R = 25$; 2) two-dimensional estimator [9] using only pilots according to $M_p = 32$, $S_p = 3$, $R = 25$; 3) one-dimensional estimator [9] using both pilots and data; 4) proposed two-dimensional estimator using only pilots according to $M_p = 10$, $S_p = 3$, $R = 25$; 5) proposed two-dimensional estimator using both pilots and data; and 6) classical linear MMSE estimator using both pilots and data referring to (1).

Choice of M_p and S_p follows (5) for estimators 1) and 4), while no sampling along frequency dimension has been considered for estimator 2). The pilot-to-symbols ratio for the estimators 1) to 3) is $7.3 \cdot 10^{-3}$, $23.4 \cdot 10^{-3}$, and $7.3 \cdot 10^{-3}$, respectively. It is apparent how the proposed two-dimensional estimator 4) outperforms the one-dimensional estimator for both cases 1) and 2). Those behaviors correspond to performance achieved at the first iteration of a turbo receiver. Estimators 3) and 5) represent the best achievable performance when feedback from decoders is available. In the last iteration of a turbo receiver, the proposed two-dimensional estimator may present a 5 dB saving with respect to the one-dimensional estimator. The classical MMSE receiver performs very poorly: the number of variables to be estimated in the model (1) is much larger than the available data due to the channel variation along both time and frequency.

V. Conclusion

A two-dimensional channel estimator for MIMO-OFDM systems has been designed in order to exploit jointly time and frequency correlations of the wireless channel via use of a Slepian expansion. Performance in terms of RMMSE-versus-SNR have been analyzed for the case in which only pilots (resp. both pilots and data) are available at the receiver, corresponding to the first (resp. last) iteration of a turbo receiver. Computer simulation shows how the proposed two-dimensional estimator outperforms the estimator based on a one-dimensional Slepian expansion, though the complexity of the former is higher than the latter. The rectangular support of the scattering function represents a worst-case scenario in which delay and Doppler are assumed independent, and detailed knowledge on the cross-correlation between delay and Doppler (i.e., effective two-dimensional support) allow to reduce the complexity further.

Appendix

Denote $\boldsymbol{A}_\psi \boldsymbol{r}$ the linear estimate for the vector of Slepian coefficients, and $\mathcal{J}_c(\boldsymbol{A}) = \mathbb{E}\{|\boldsymbol{\psi} - \boldsymbol{A}\boldsymbol{r}|^2\}$ the cost function. The estimator is found as

$$\boldsymbol{A}_\psi = \mathbb{E}\{\boldsymbol{\psi}\boldsymbol{r}^{\mathrm{H}}\} \left(\mathbb{E}\{\boldsymbol{r}\boldsymbol{r}^{\mathrm{H}}\}\right)^{-1}. \tag{6}$$

From (3), $\mathbb{E}\{\boldsymbol{\psi}\boldsymbol{r}^{\mathrm{H}}\} = \mathbb{E}\{\boldsymbol{\psi}(\boldsymbol{\Xi}\boldsymbol{\psi} + \boldsymbol{w})^{\mathrm{H}}\} = \boldsymbol{C}_\psi \hat{\boldsymbol{\Xi}}^{\mathrm{H}}$ and $\mathbb{E}\{\boldsymbol{r}\boldsymbol{r}^{\mathrm{H}}\} = \mathbb{E}\{(\boldsymbol{\Xi}\boldsymbol{\psi} + \boldsymbol{w})(\boldsymbol{\Xi}\boldsymbol{\psi} + \boldsymbol{w})^{\mathrm{H}}\} = \mathbb{E}\{\boldsymbol{\Xi}\boldsymbol{C}_\psi\boldsymbol{\Xi}^{\mathrm{H}}\} + \sigma_w^2 \boldsymbol{I}_{(NMS)}$. The diagonal structure of $\boldsymbol{C}_\psi$ is due to the independence of channels among different transmit antennas and/or receive antennas, and to the orthogonality of the DPS sequences

$$\mathbb{E}\left\{\psi_{n,k}[\ell,i]\psi^*_{n',k'}[\ell',i']\right\} = \frac{\lambda_\ell^{(\mathrm{d})}}{2\eta_{\max}^{(\mathrm{d})}} \frac{\lambda_i^{(\mathrm{D})}}{2\nu_{\max}^{(\mathrm{D})}} \delta_{n,n'}\delta_{k,k'}\delta_{\ell,\ell'}\delta_{i,i'}.$$

The independence of transmit antennas and, due to the effect of random interleaving, also of OFDM blocks, gives

$$\mathbb{E}\left\{x_k[m,s]x^*_{k'}[m',s']\right\} = \begin{cases} 1, & \text{if } k = k', m = m', s = s' \\ \hat{x}_k[m,s]\hat{x}^*_{k'}[m',s'], & \text{else.} \end{cases}$$

Some algebra shows that $\mathbb{E}\{\boldsymbol{\Xi}\boldsymbol{C}_\psi\boldsymbol{\Xi}^{\mathrm{H}}\} = \hat{\boldsymbol{\Xi}}\boldsymbol{C}_\psi\hat{\boldsymbol{\Xi}}^{\mathrm{H}} + \boldsymbol{\Theta}$. Finally, appropriate substitutions into (6) furnish the estimator $\boldsymbol{A}_\psi = \boldsymbol{C}_\psi\hat{\boldsymbol{\Xi}}^{\mathrm{H}}(\hat{\boldsymbol{\Xi}}\boldsymbol{C}_\psi\hat{\boldsymbol{\Xi}}^{\mathrm{H}} + \boldsymbol{\Delta})^{-1}$, and then (4).

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